Isochronism of Simple Pendulum
Relationship between velocity of pendulum and the law of conservation of mechanical energy

NaRiKa Corporation

Isochronism of Simple Pendulum
Galileo's discovery in 16th century

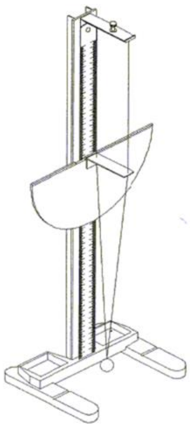
1. Historical Background

Galileo Galilei, later called "Father of science", was born in Tuscany, Italy over 400 years ago. Some anecdotes are told about him when he was a mathematics professor at the University of Pisa as follows.

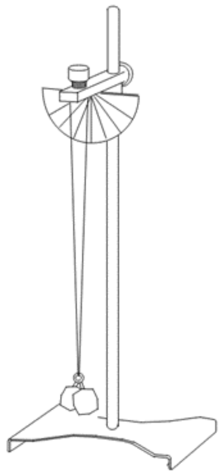
In the latter half of 16th century, Galileo discovered the crucial property that makes pendulums useful as timekeepers, called the isochronism of simple pendulum (pendulum), when watching the swings of the bronze chandelier in the cathedral of Pisa (photo on the right) using his pulse as a timer. The property is that period of pendulum swing is approximately independent of its amplitude or width of the swing. On another occasion, Galileo dropped balls (a bullet and a cannonball) of the same material, but different masses, from the Leaning Tower of Pisa, in the presence of professors from the University of Pisa, to demonstrate that their time of descent was the same and independent of their mass. Based on these two anecdotes, Galileo derived the two laws, by one of which freefall bodies of different masses have the same free-fall duration time and, by the other of which the distance traveled by a free fall body is proportional to the square of the elapsed time. This teachers' guide is intended to verify the Galileo's law of isochronism of pendulum by using Narika's equipment for pendulum experiments.

2. Equipment for pendulum experiments

Shown below is equipment for pendulum experiments manufactured and sold by Narika Corporation as it's representative product for school education in general. Equipment for teachers' demonstration is C15-4474. Equipment for experiments by group of students is C15-4475. Below are examples of experiments and data results using C15-4475.



C15-4474 Equipment for pendulum experiments FP



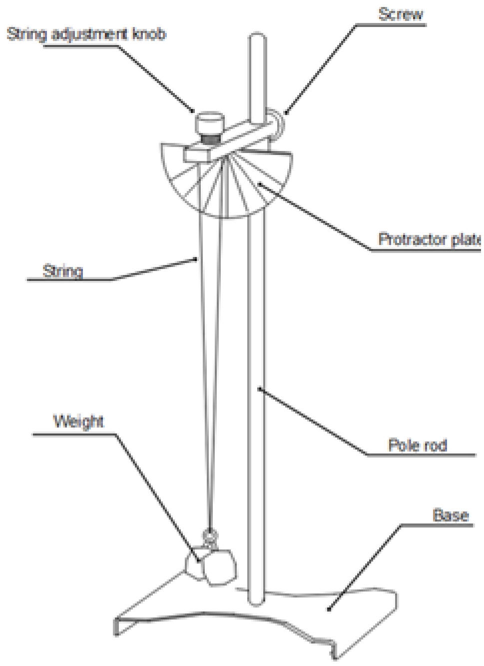
C15-4475 Pendulum

3. C15-4475 Pendulum set

Regarding demonstration of the law of isochronism of pendulum, it is required to verify the correlation among experimentally obtained values of "length of string", "weight of mass", "amplitude (swing width) of pendulum" and "cycle". To do this inevitably requires to repeat experiment many times to statistically derive the correlation despite time constraint of teachers who normally do not have enough time to collect sufficient data. Because of this background, a lot of teachers desired an apparatus for pendulum experiments with shorter setup time for assembling/adjustment/cleaning-up, as well as, user-friendliness and repeatability for their experiment.

Essential product used in this teachers' guide is Narika's "Pendulum experiment set". It is equipped with the wind-up mechanism for string storage that enables easier adjustment of string's length, and prevention of non-linear swing of weight due to the V-shaped pendulum string. This product was designed in consideration of better space-saving storage and easier string replacement based on teachers' demands.

1. Specification and Contents



1) Pole rod

Material: Brass and nickel chrome plate

Size: ϕ12×440mm (ca)

2) Base

Material: Steel

Size: 180×140×15mm

Weight: ca. 480g

3) Protractor plate

Material: Plastics

Shape: Semi-circular (10 degrees steps max.120)

Size ϕ100mm

4) String adjustment knob (system):

String reeling knob

5) Screw:

Screw fitting with rod, M4×8mm

6) Thread

Length: max. 1m, to be suspended in V-shape

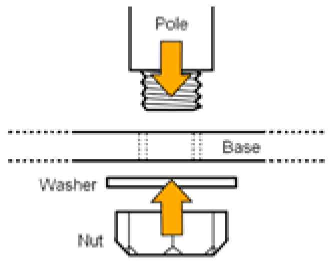
7) Weights

Weight: 10g×10 pieces

Material: Plastics

2. Preparation for the experiment

1) Assembling Stand



① Remove the nut and washer from the pole at once.

② Install the pole into the hole in the base.

③ Tighten the pole by the washer and the nut using tools.

④ Before experiment, check that the screw is tightened enough.

2) How to set up Pendulum

① Loose its screw and turn the protractor plate of the pendulum unit to be mounted orthogonally on its rod (see step1).

② Tighten the screw on the rod at the orthogonal position of the protractor plate and the rod (see step2).

③ Protractor plate should be fixed by the screw (see step3).

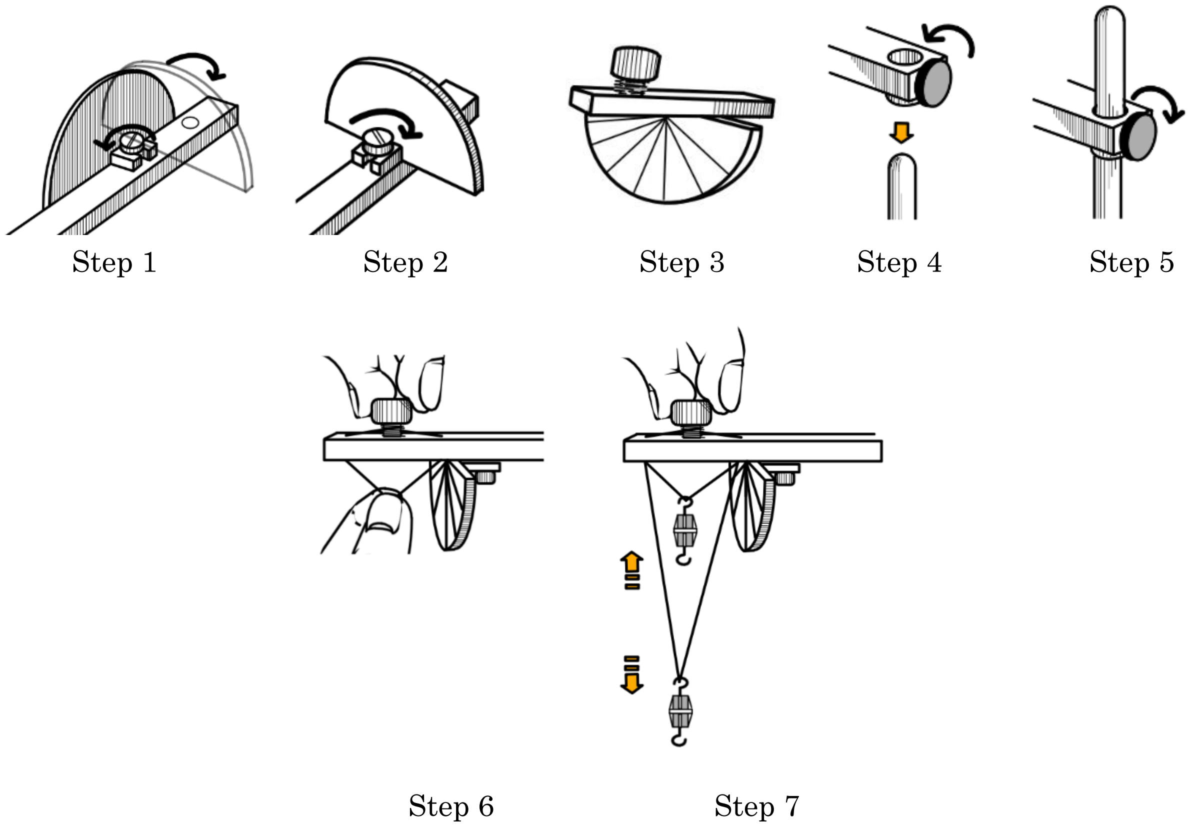
④ Loose the screw at the end of the rod and insert the top of pole (see step4).

⑤ Tighten the screw at a suitable position (height) for the pendulum experiment (see step5).

⑥ Pull out a string while turning the string adjustment knob anticlockwise by your fingers (see step6).

⑦ 7. Hook a weight on the string.

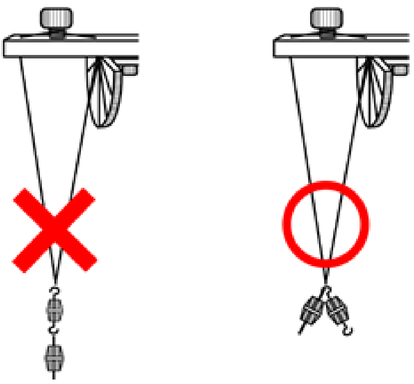
⑧ 8. Adjust the string to suitable length for the experiment by turning the string adjustment knob clockwise or anticlockwise (see step7).



3) Caution on Pendulum Apparatus during the experiment

① Position of weights

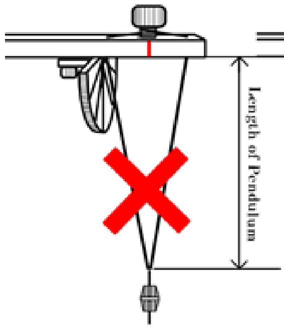
If the weights are hooked as shown in Fig.1, they may cause errors in the experiment. When using several weights in the experiment, hook them as shown in Fig.2 in order to align the center of weights (see Fig.1 & 2).



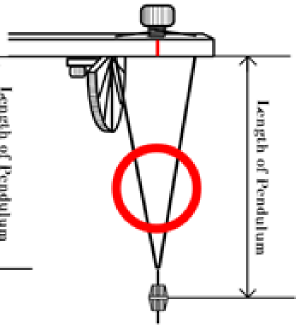
[Fig.1] [Fig.2]

② Length of Pendulum (thread length)

Common mistake in measuring the length of pendulum is measuring the distance between the bottom vertex of a triangle, where weight is hooked (V shape), and the bottom surface of the rod (see Fig.3). The correct length of pendulum is from the center of weight to the bottom surface of the rod – red line (see Fig.4). You should use the vertical line distance shown in Fig.4 as the correct length of pendulum.



[Fig.3]



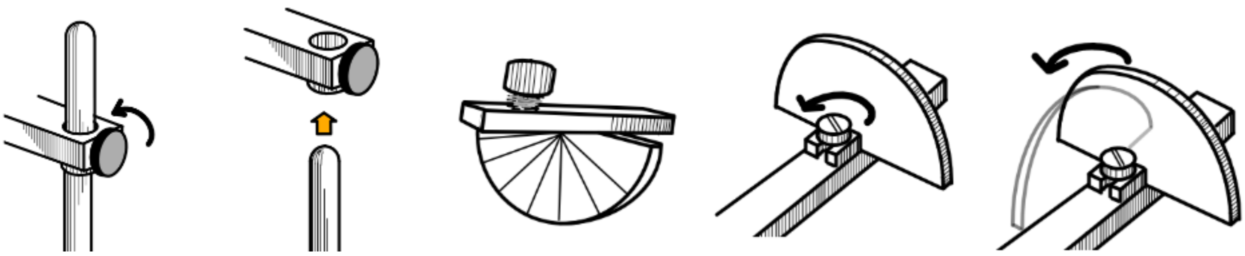
[Fig.4]

4) Storing Pendulum Apparatus

The pendulum apparatus (unit) has storage mode.

① Pull the pendulum unit out from the pole rod (see step 1, 2, 3).

② Turn the Protractor plate by 90 degrees (see step 4, 5).



Step 1 Step 2 Step 3 Step 4 Step 5

5) Maintenance of Pendulum

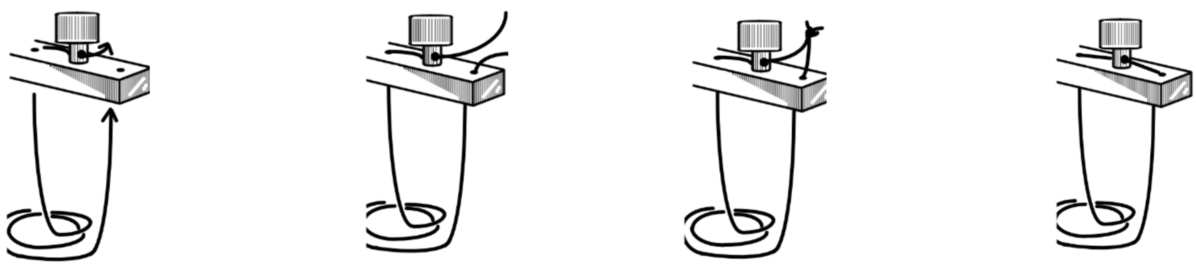
When replacing a thread of pendulum to a new one, follow steps below.

① Remove existing string from the pendulum unit.

② Insert a new string into 2 holes, one edge on the top of the pendulum unit and the other edge on the string adjustment knob (see step 1 and step 2).

③ Tie both ends of the string (see step 3).

④ Move the tied knot into the hole of the string adjustment knob (see step 4).



Step 1 Step 2 Step 3 Step 4

4. Relationship between velocity of pendulum and the law of conservation of mechanical energy

Isochronism of simple pendulum has been commonly and traditionally proved through measurement of the required time for each cycle. On the other hand, as shown in preceding section “3. Isochronism of Simple Pendulum ~Measurement of pendulum velocity~”, we have certainly realized how easy it is to measure velocity of pendulum by using portable speed measurement device “BeeSpi v”. Therefore, in this section, we will investigate and verify the relationship between motion of pendulum and the law of conservation of mechanical energy by directly measuring the velocity using BeeSpi v. Formula (1) shown below can be derived when all the potential energy is converted to the kinetic energy at the lowest point of the pendulum motion. By solving formula (1) for v, we can obtain equation (2) and (3).

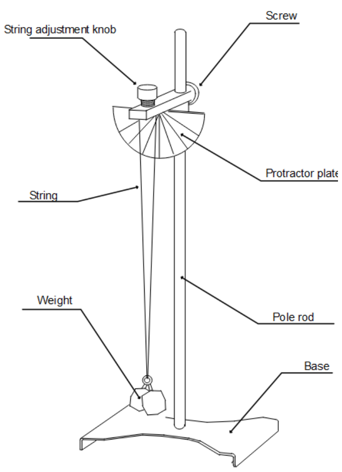
$$\frac{1}{2}mv^2 = mgh \quad \text{---- (1)}$$

$$v = \sqrt{2gh} \qquad v^2 = 2gh \quad \text{--- (2) (3)}$$

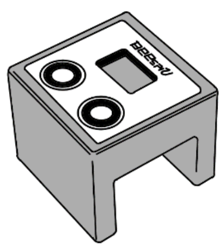
Based on above equations, we can directly measure velocity of the pendulum at the lowest point (B) after being released from the highest point (A) by using BeeSpi v. Compare the measured values with the theoretical values derived from the formulas above to investigate the difference.

1. Necessary equipment

C15-4475 Pendulum set: 1 (weight and stand included)
S77-1321 BeeSpi v (Portable speed measurement device): 1
Ruler (around 50 cm long): 1



Pendulum experiment set

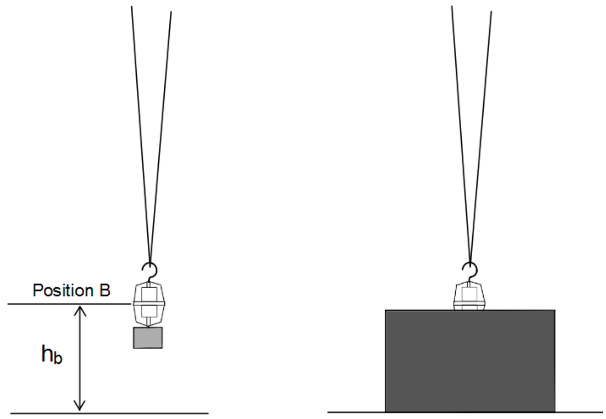


BeeSpi v

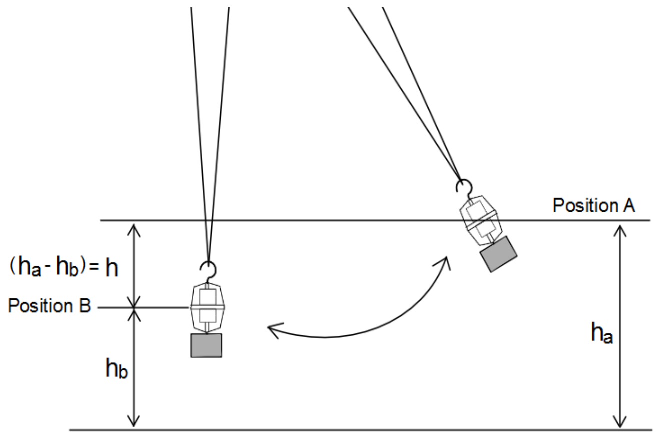
2. How to make experiment

Measure the height of the gravity center of weight (pendulum) in a resting state from the surface of laboratory bench (Position B, h_b). Note that the height of the weight shall be adjusted to fully shield built-in photogates of the BeeSpi v because measurement would not be successfully carried out if the position of the weight is higher than the photogates.

It is recommended to attach a small slip of paper wrapping around the lower hook to make sure the BeeSpi v securely measures velocity.



Calculate the difference in height between start point of releasing the weight (Position A, h_a) and (Position B, h_b) to obtain height ($h = h_a - h_b$) of the pendulum motion. Measure the speed of pendulum at position B after releasing it from position A with the BeeSpi v by increasing the height (h) at 5cm (0.05m) intervals. Measure velocity of the pendulum at Position B at least five times consecutively in each setting of the height (h) to get the average and to obtain five results of average velocity.



3. Setup of BeeSpi v

1) Insert two AAA batteries in BeeSpi v after sliding off the battery cover.

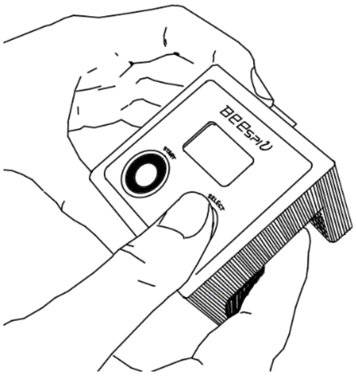
2) Check if four numeric characters, "0" (zero), appears on the LCD display. In case of using BeeSpi v with batteries already inserted, you may find nothing appears on the LCD display, which means it is off. If so, turn the power on by pressing "START" or "SELECT" button.

3) Measurement unit appears at the right edge of the LCD display. Unit of "m/s" is selected for every experiment covered in this teachers' guide. To select the measurement unit, press "SELECT" button for more than two seconds to change the unit in order of "m/s" → "m/h" → "cm/s".

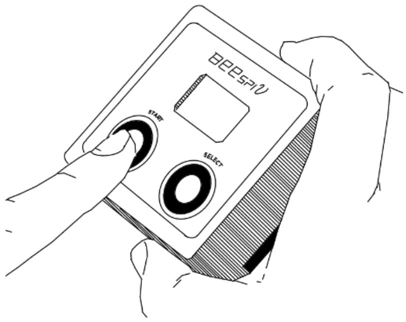
4) Press "START" button to set BeeSpi v into measurement mode after confirming "m/s" unit is appearing on the LCD display.

5) In the measurement mode, BeeSpi v is activated if the selected unit "m/s" is flashing. In this state, speed measurement starts when the first photogate is shielded and ends when the second photogate is shielded by the moving object running through the BeeSpi v.

6) BeeSpi v retains up to five latest measurement results that can be brought up with data number (1~5) at the upper-left of the LCD display by repeatedly pressing "SELECT" button.



SELECT Button:
To activate measurement mode



START Button:
To select unit and retrieve measurement results

4. Results of experiment

Pendulum velocity measurements were made using BeeSpi v under the following conditions: length of (the string of) pendulum (L) was fixed at 0.40m and weight 0.01kg were used as shown in Table 1. Then, pendulum velocity was calculated in each setting of height (h) based on the equations below and both results were shown graphically in Figure 2. This indicates that the measured values are matching the theoretical ones if the height (h) is 0.25m or less.

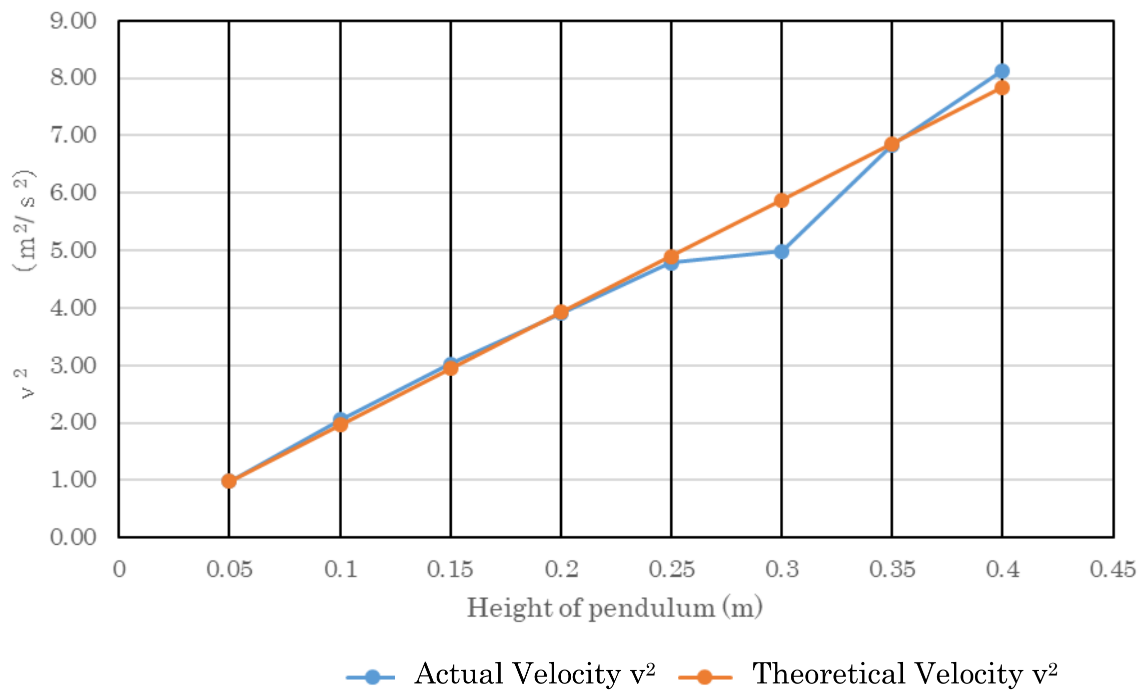
$$v = \sqrt{2gh} \qquad v^2 = 2gh$$

Table 1. Height (h) and velocity of pendulum

L: 0.40m, m: 0.01Kg

Height (m)			Velocity at Position B (m/s)						v^2	Theoretical velocity (v_t) (m/s)	v_t^2
Position A (ha)	Position B (hb)	(ha - hb)	1	2	3	4	5	Average velocity (v)			
0.10	0.05	0.05	0.98	0.99	0.96	1.01	1.01	0.99	0.98	0.99	0.98
0.15	0.05	0.10	1.47	1.44	1.42	1.44	1.39	1.43	2.05	1.40	1.96
0.20	0.05	0.15	1.77	1.71	1.74	1.77	1.70	1.74	3.02	1.71	2.94
0.25	0.05	0.20	1.98	2.00	1.96	1.99	1.95	1.98	3.90	1.98	3.92
0.30	0.05	0.25	2.19	2.21	2.23	2.15	2.15	2.19	4.78	2.21	4.90
0.35	0.05	0.30	2.42	2.21	2.23	2.15	2.15	2.23	4.98	2.42	5.88
0.40	0.05	0.35	2.59	2.64	2.53	2.69	2.62	2.61	6.83	2.62	6.86
0.45	0.05	0.40	2.83	2.88	2.84	2.83	2.88	2.85	8.13	2.80	7.84

Figure 2. Relationship between height of pendulum and squared velocity



5. Relationship between velocity of pendulum and the law of conservation of mechanical energy (Part II)

As investigated in section 4, applicability of the law of conservation of mechanical energy was examined through pendulum motion experiments. In this section will be verified that potential energy and kinetic energy are exchangeable with each other and the total amount of both types of energy is constant by using the Pendulum apparatus for verification of the “Law of Conservation of Mechanical Energy” by experiment.

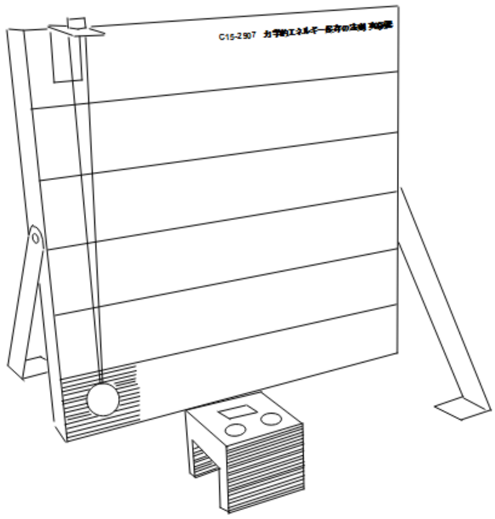


Figure on the left is product developed by Narika called “Apparatus for law of mechanical energy conservation (C15-2907)” that comes with BeeSpi v and specially designed pendulum.

Unlike the traditional tape timer, BeeSpi v can directly measure velocity of an object, which makes it possible for students to derive the correlation between potential energy and kinetic energy based on the measured results.

Nevertheless, teachers should guide students to spontaneously generalize the measured results.

1. Experiment setup

1) Stretch its support legs on both sides of the apparatus on laboratory bench to retain the apparatus in a vertical position.

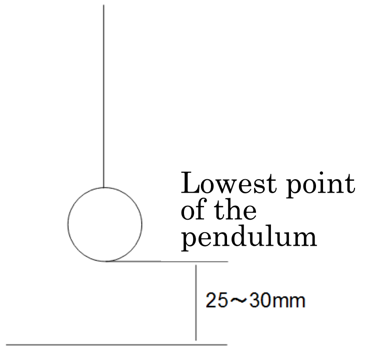
2) Adjust height of the lowest point of the pendulum that comes with the apparatus between 25mm and 30mm from the laboratory bench.

3) Place BeeSpi v up-side down beside the gray mark on the apparatus and adjust the position of the BeeSpi v so that it is located directly beneath the pendulum. Adjust the setting so that the pendulum does not collide with the BeeSpi v when swinging. Again, make sure the pendulum fully shields the photogate sensors of the BeeSpi v.

4) Practice release of the pendulum using a ruler or a board by swiftly removing the ruler or board at any intended height after bringing up the pendulum by the ruler or board.

2. Experiment

1) Record the height in meters between the lowest point of the pendulum and the surface of the laboratory bench.



2) Set BeeSpi v in measurement mode by pushing START button and place it up-side down on the laboratory bench beside the gray mark paying attention that nothing would shield the photogate sensors while keeping the pendulum in your hand away from the BeeSpi v.

3) Bring up the pendulum slowly by using the ruler until the bottom of the pendulum comes to (is aligned with) one of the lines on the apparatus (50mm intervals).

4) Swiftly remove the ruler away from the pendulum.

5) Record the measured values displayed on the BeeSpi v.

6) Repeat above 2 to 5 at least five times at each height to measure the velocity of the pendulum.

3. Result

Summary of the measurement results are shown in Table 2. In Figure 3, a non-linear graph shows the relationship between actual distance from the initial height to the lowest point of the weight and the average velocity of the weight at the lowest point of the pendulum. Equation, $y = 0.0479x^2 + 0.0019x$, can be provided when generating an approximate curve with MS-Excels' function. Therefore, the measurement results are shown by a quadratic function.

Note that it is strongly recommended to process the measured data using PC (MS-Excel) to prevent students from mistakenly making linear function graph based on the measured results plotted on graph paper using their rulers.

Next step is to graphically show the correlation between squared average velocity and actual fall distance of the weight. Linear function graph, $y = 0.049x$, can be provided when generating an approximate curve with MS-Excel function. This equation is to show the relationship between x and y, given x is the squared average velocity (v^2) and y is the actual fall distance of the weight (h). Consequently, this equation can be transformed to:

$$h = 0.049v^2$$

Relationship between potential energy and kinetic energy can be mathematically expressed as follows: "$mgh = 1/2mv^2$". Then, we can substitute "h" into this relationship and get equation below.

$$mgh = 0.049mgv^2 = 1/2mv^2$$

This equation represents state when all of the potential energy of the pendulum has converted to the kinetic energy. In other words, mechanical energy equals the sum of potential energy and kinetic energy.

Note: the coefficients in linear function and quadratic function derived from the measured results do not matter much, but the graph based on the measured results are more important.

Thus, traditional tape timers are not suitable for experiments covered in this teachers' guide because they are not intended to measure velocity of objects directly. On the other hand, the combination of "BeeSpi v" (portable speed measurement device) with the product covered in this section will be easy-to-use and more effective to conduct such experiments.

Table 2. Results of experiment

Height of the falling point of weight [m]	Actual falling height (fall distance) of weight [m]	Velocity of the weight at the lowest point of the pendulum [m/s]					Average velocity (v) [m/s]	Squared average velocity (v^2) $[m/s]^2$
		1st measurement	2nd measurement	3rd measurement	4th measurement	5th measurement		
0.050	0.020	0.63	0.64	0.65	0.63	0.64	0.64	0.41
0.100	0.070	1.19	1.20	1.18	1.18	1.18	1.19	1.41
0.150	0.120	1.57	1.56	1.53	1.54	1.56	1.55	2.41
0.200	0.170	1.85	1.92	1.87	1.86	1.88	1.88	3.52
0.250	0.220	2.11	2.11	2.13	2.12	2.12	2.12	4.49

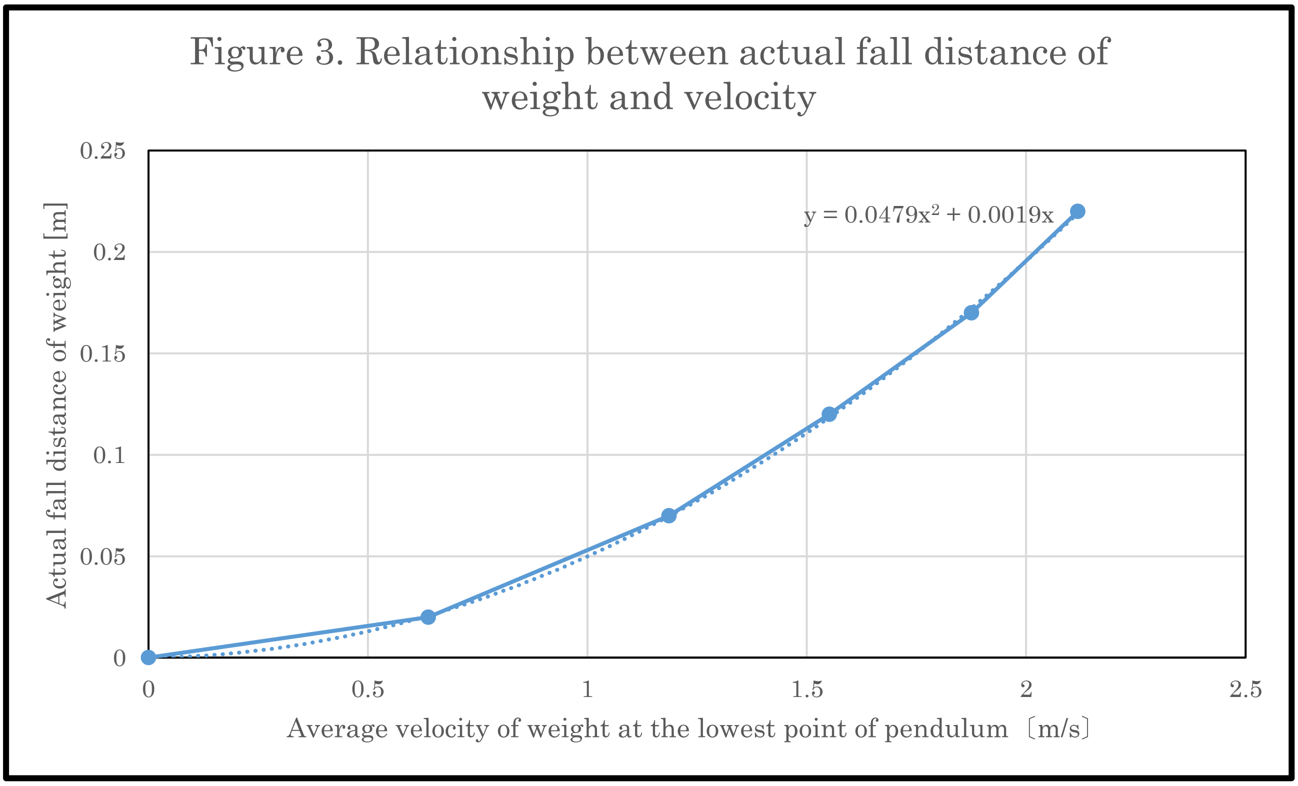


Figure 3. Relationship between actual fall distance of weight and velocity
y = 0.0479x² + 0.0019x
Actual fall distance of weight [m]
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5
Average velocity of weight at the lowest point of pendulum 〔m/s〕

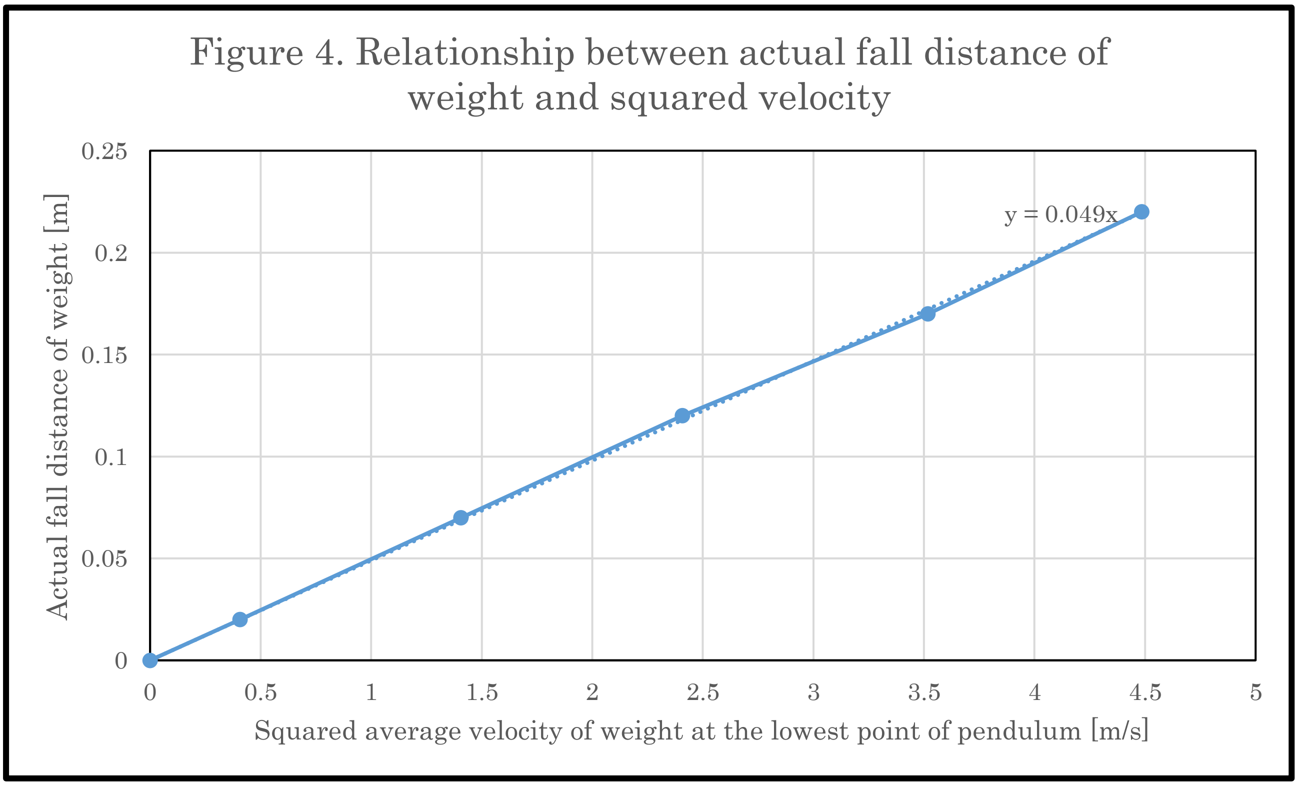


Figure 4. Relationship between actual fall distance of weight and squared velocity
y = 0.049x
Actual fall distance of weight [m]
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Squared average velocity of weight at the lowest point of pendulum [m/s]

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