Isochronism of Simple Pendulum

Investigation of time required for a periodical swinging motion of pendulum

NaRiKa Corporation

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Galileo's discovery in 16th century

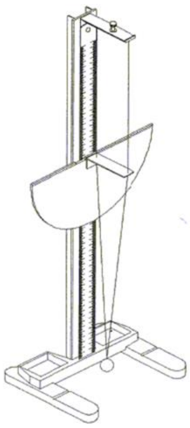
1. Historical Background

Galileo Galilei, later called "Father of science", was born in Tuscany, Italy over 400 years ago. Some anecdotes are told about him when he was a mathematics professor at the University of Pisa as follows.

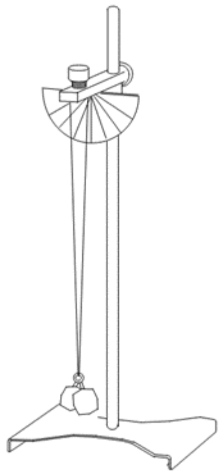
In the latter half of 16th century, Galileo discovered the crucial property that makes pendulums useful as timekeepers, called the isochronism of simple pendulum (pendulum), when watching the swings of the bronze chandelier in the cathedral of Pisa (photo on the right) using his pulse as a timer. The property is that period of pendulum swing is approximately independent of its amplitude or width of the swing. On another occasion, Galileo dropped balls (a bullet and a cannonball) of the same material, but different masses, from the Leaning Tower of Pisa, in the presence of professors from the University of Pisa, to demonstrate that their time of descent was the same and independent of their mass. Based on these two anecdotes, Galileo derived the two laws, by one of which freefall bodies of different masses have the same free-fall duration time and, by the other of which the distance traveled by a free fall body is proportional to the square of the elapsed time. This teachers' guide is intended to verify the Galileo's law of isochronism of pendulum by using Narika's equipment for pendulum experiments.

2. Equipment for pendulum experiments

Shown below is equipment for pendulum experiments manufactured and sold by Narika Corporation as it's representative product for school education in general. Equipment for teachers' demonstration is C15-4474. Equipment for experiments by group of students is C15-4475. Below are examples of experiments and data results using C15-4475.



C15-4474 Equipment for pendulum experiments FP



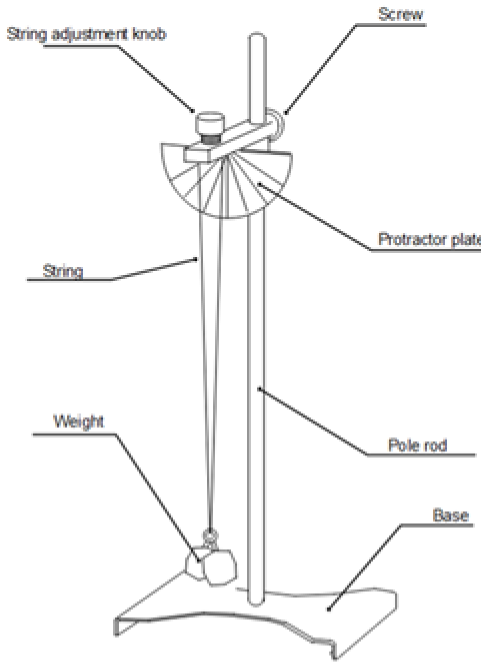
C15-4475 Pendulum

3. C15-4475 Pendulum set

Regarding demonstration of the law of isochronism of pendulum, it is required to verify the correlation among experimentally obtained values of "length of string", "weight of mass", "amplitude (swing width) of pendulum" and "cycle". To do this inevitably requires to repeat experiment many times to statistically derive the correlation despite time constraint of teachers who normally do not have enough time to collect sufficient data. Because of this background, a lot of teachers desired an apparatus for pendulum experiments with shorter setup time for assembling/adjustment/cleaning-up, as well as, user-friendliness and repeatability for their experiment.

Essential product used in this teachers' guide is Narika's "Pendulum experiment set". It is equipped with the wind-up mechanism for string storage that enables easier adjustment of string's length, and prevention of non-linear swing of weight due to the V-shaped pendulum string. This product was designed in consideration of better space-saving storage and easier string replacement based on teachers' demands.

1. Specification and Contents



1) Pole rod

Material: Brass and nickel chrome plate

Size: ϕ12×440mm (ca)

2) Base

Material:	Steel
Size:	180×140×15mm
Weight:	ca. 480g

3) Protractor plate

Material:	Plastics
Shape:	Semi-circular (10 degrees steps max.120)

Size ϕ100mm

4) String adjustment knob (system):

String reeling knob

5) Screw:

Screw fitting with rod, M4×8mm

6) Thread

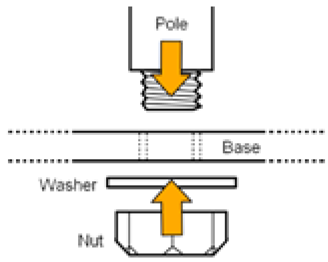
Length: max. 1m, to be suspended in V-shape

7) Weights

Weight:	10g×10 pieces
Material:	Plastics

2. Preparation for the experiment

1) Assembling Stand



① Remove the nut and washer from the pole at once.

② Install the pole into the hole in the base.

③ Tighten the pole by the washer and the nut using tools.

④ Before experiment, check that the screw is tightened enough.

2) How to set up Pendulum

① Loose its screw and turn the protractor plate of the pendulum unit to be mounted orthogonally on its rod (see step1).

② Tighten the screw on the rod at the orthogonal position of the protractor plate and the rod (see step2).

③ Protractor plate should be fixed by the screw (see step3).

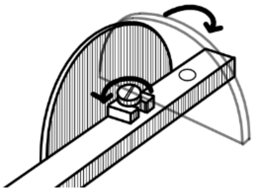
④ Loose the screw at the end of the rod and insert the top of pole (see step4).

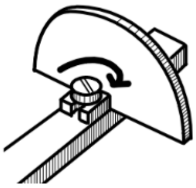
⑤ Tighten the screw at a suitable position (height) for the pendulum experiment (see step5).

⑥ Pull out a string while turning the string adjustment knob anticlockwise by your fingers (see step6).

⑦ 7. Hook a weight on the string.

⑧ 8. Adjust the string to suitable length for the experiment by turning the string adjustment knob clockwise or anticlockwise (see step7).


Step 1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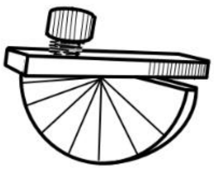
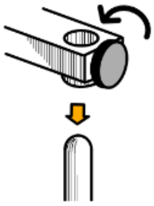
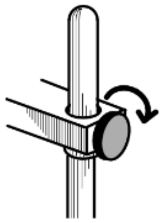
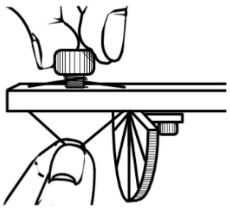
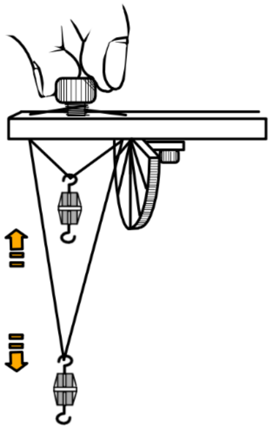

Step 3


Step 4


Step 5

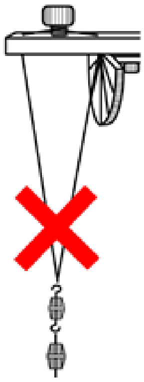

Step 6

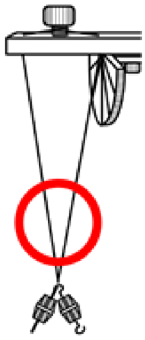

Step 7

3) Caution on Pendulum Apparatus during the experiment

① Position of weights

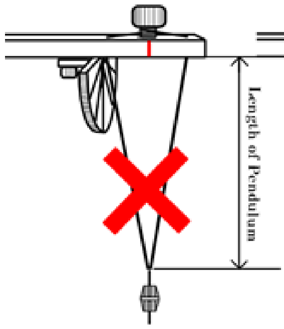
If the weights are hooked as shown in Fig.1, they may cause errors in the experiment. When using several weights in the experiment, hook them as shown in Fig.2 in order to align the center of weights (see Fig.1 & 2).


[Fig.1]

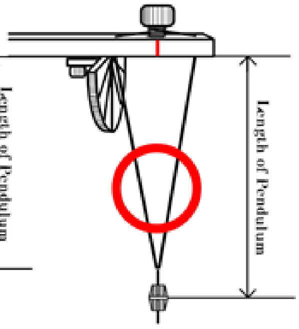

[Fig.2]

② Length of Pendulum (thread length)

Common mistake in measuring the length of pendulum is measuring the distance between the bottom vertex of a triangle, where weight is hooked (V shape), and the bottom surface of the rod (see Fig.3). The correct length of pendulum is from the center of weight to the bottom surface of the rod – red line (see Fig.4). You should use the vertical line distance shown in Fig.4 as the correct length of pendulum.



[Fig.3]



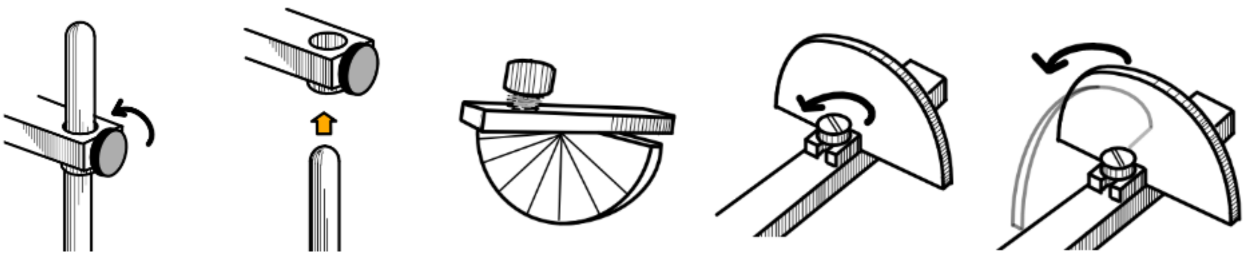
[Fig.4]

4) Storing Pendulum Apparatus

The pendulum apparatus (unit) has storage mode.

① Pull the pendulum unit out from the pole rod (see step 1, 2, 3).

② Turn the Protractor plate by 90 degrees (see step 4, 5).



Step 1 Step 2 Step 3 Step 4 Step 5

5) Maintenance of Pendulum

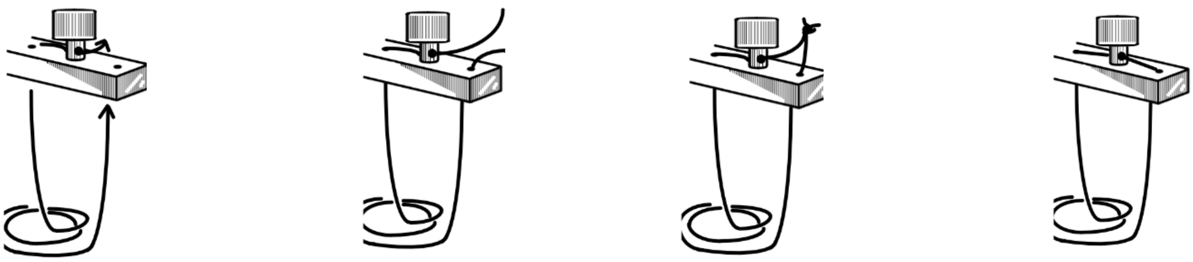
When replacing a thread of pendulum to a new one, follow steps below.

① Remove existing string from the pendulum unit.

② Insert a new string into 2 holes, one edge on the top of the pendulum unit and the other edge on the string adjustment knob (see step 1 and step 2).

③ Tie both ends of the string (see step 3).

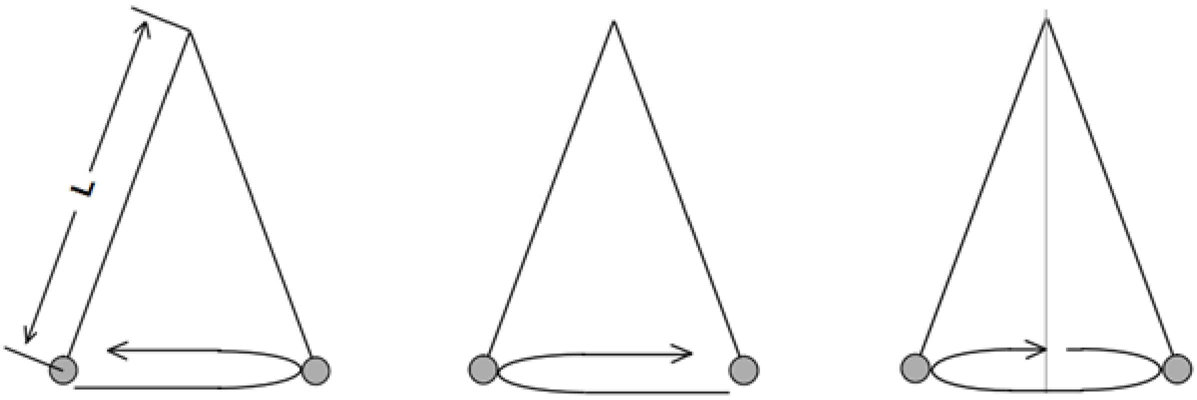
④ Move the tied knot into the hole of the string adjustment knob (see step 4).



Step 1 Step 2 Step 3 Step 4

1. How to make experiment

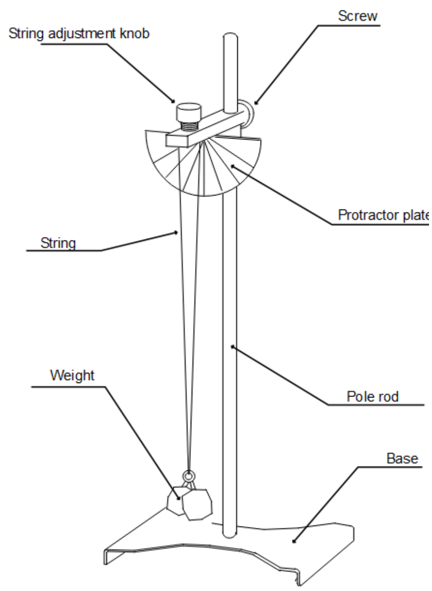
To verify the isochronism of simple pendulum through experiments, the correlation among the essential values of “[1] pendulum length”, “[2] the mass of weight of the pendulum” and “[3] amplitude of the pendulum” has to be experimentally examined. In doing so, it should be noted that correct length of pendulum means length from the center of weight to the fulcrum point (see the figure A below). When measuring how much time it takes for each swing of pendulum, it is recommended to do so based on one of the figures below A, B or C. However, elementary school students are to be advised to use A or B as easier way of experiment. In order to measure effectively how much time it takes for weight to complete one swing (cycle), it is recommended to measure the amount of time required for consecutive ten (10) swings (cycles) in either of the pattern A or B by using a stopwatch and divide it by ten (10) to obtain the time required for one swing (cycle).



A:one swing (round-trip) B: one swing (round-trip) C: one cycle

2. Necessary equipment

C15-4475 Pendulum set x1 (weight and stand included)
A05-5214 Stopwatch x1



3. Pendulum cycle measurement 1:

Result of measurement of pendulum cycles with different mass

Pendulum cycles measurement were made under the following conditions: length of (the string of) pendulum (L) was fixed at 25cm, amplitude fixed at 5 degrees/10 degrees and weights 10g, 20g, 30g and 40g were used respectively. Results of the experiment shows that the cycle (amount of the time required for one swing) is within the range of 1.00 sec and 1.02 sec regardless of the mass of the weight.

Table 1. Result of measurement of pendulum cycles with different mass
L=0.25m, Amplitude: 5 degrees

Weight (g)	Time required for consecutive 10 swings (sec)					Average time for 10 swings (sec)	Average time for 1 swing (sec)
10	9.88	10.14	10.04	10.07	9.95	10.02	1.00
20	10.08	9.93	9.97	9.94	10.01	9.99	1.00
30	10.12	9.91	9.89	10.04	10.05	10.00	1.00
40	10.25	10.25	10.09	10.10	10.20	10.18	1.02

Table 2. Result of measurement of pendulum cycles with different mass
L=0.25m, Amplitude: 10 degrees

Weight (g)	Time required for consecutive 10 swings (sec)					Average time for 10 swings (sec)	Average time for 1 swing (sec)
10	10.13	10.15	10.22	10.05	10.01	10.11	1.01
20	10.03	10.08	10.14	10.04	10.03	10.06	1.01
30	10.09	10.05	9.98	9.99	9.98	10.02	1.00
40	10.19	10.14	10.16	10.25	10.28	10.20	1.02

4. Pendulum cycle measurement 2

Result of measurement of pendulum cycles with different amplitudes

Pendulum cycles measurement were made under the following conditions: length of (the string of) pendulum (L) was fixed at 25cm, weight fixed at 10g and amplitudes of different degrees were used. Results of the experiment shows that the cycle gets longer when the amplitude becomes over 35 degrees. The cycle is constant within the range of 1.00 and 1.02 sec when the amplitudes are in the range from 5 to 30 degrees. This means isochronism of a pendulum is more obvious within the amplitude range of 15 degree or less. The most favorable condition for isochronism can be achieved when the amplitude is 5 degrees or less.

Table 3. Result of measurement of pendulum cycles with different amplitudes

L=0.25m, Mass of weight: 10g

Amplitudes	Time required for consecutive 10 swings (sec)					Average time for 10 swings (sec)	Average time for 1 swing (sec)
5^o	9.88	10.14	10.04	10.07	9.95	10.02	1.00
10^o	10.13	10.15	10.22	10.05	10.01	10.11	1.01
15^o	10.23	10.21	9.97	10.06	10.07	10.11	1.01
20^o	10.22	10.23	10.11	10.19	10.16	10.18	1.02
25^o	10.06	10.11	10.15	10.21	10.04	10.11	1.01
30^o	10.09	10.15	9.95	10.23	10.25	10.13	1.01
35^o	10.31	10.27	10.21	10.21	10.13	10.23	1.02
40^o	10.30	10.25	10.31	10.25	10.33	10.29	1.03
50^o	10.55	10.51	10.41	10.39	10.39	10.45	1.05
60^o	10.61	10.71	10.53	10.66	10.65	10.63	1.06

5. Pendulum cycle measurement 3

Result of measurement of pendulum cycles with different string length

Pendulum cycles measurement were made under the following conditions: amplitude was fixed at 5 degrees, weight fixed at 10g and the string with different lengths were used. After repeating the measurement 10 times, the average amount of time was calculated. Result of the experiment shows that the cycle of pendulum (T) varies depending on the string length as shown in Table 4 and Figure 1.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

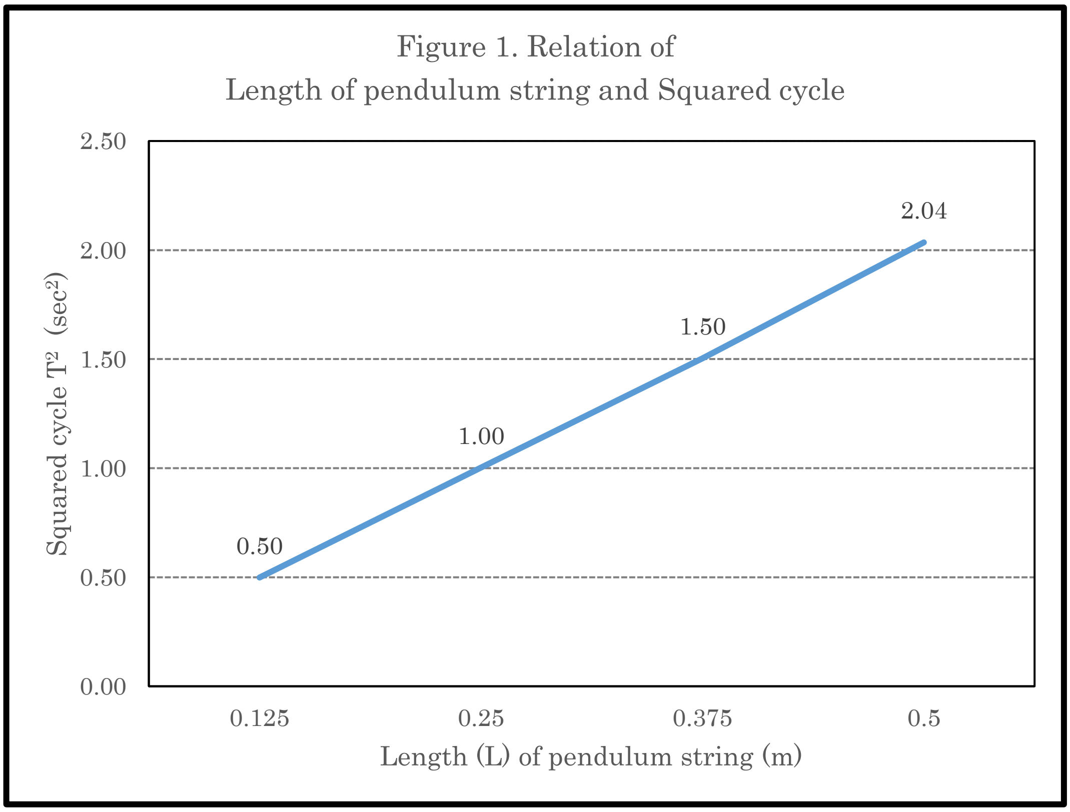
The cycle (T^2) and length (L) of a pendulum can be expressed in terms of a linear equation when squaring the both sides of the formula of cycle of pendulum. The Figure 1 shows a straight-line graph for this linear equation.

$$T^2 = 4\pi^2\frac{L}{g} = \frac{4\pi^2}{g}L$$

Table 4. Cycle (T) and squared cycle (T^2) of pendulum with different length of string

Amplitude: 5 degrees, Weight: 10g

Amplitude		Length (L) of pendulum string (m)			
		0.125	0.250	0.375	0.50
5°	Cycle (T)	0.71	1.00	1.22	1.43
	Squared cycle (T^2)	0.50	1.00	1.50	2.04



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